Bianchi Type III String Cosmological Model With Bulk Viscosity

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Abstract :

We have investigation cosmic string in Bianchi Type-III cosmological models in the presence and absence of bulk viscosity. The physical and geometric properties of model are discussed.

***Keywords**: String Cosmological , Bulk Viscosity.*

1. Introduction:

It is still a problem to know exact physical situation at the very early stage of the formation of the universe. There has been considerable interest in string cosmology because cosmic string play an important role in the study of the early universe. It appears that after the big bang, the universe may have undergone series of phase transitions as its temperature lower down. During phase transition the symmetry of the universe is broken spontaneously.
It can give rise to topologically stable defects. Among the various topological defects and before the creation of particles in the early universe, strings have interesting cosmological consequences Letelier [1] has solved Einstein's field equation for a cloud of massive string and obtained cosmological model in Bianchi I and Kantowski Sach's space-time.

Bianchi type models have been studied by several authors in an attempt to understand better the observed small amount of anisotropy in the universe. The same models have also been used to examine the role of certain anisotropic sources during the formation of the large-scale structures we see in the universe today. Some Bianchi cosmologies, for example, are natural hosts of large-scale magnetic fields and therefore, their study can shed light on the implications of cosmic magnetism for galaxy formation. The simplest Bianchi family that contains the flat FRW universe as a special case are the type-I space-times.

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic , from which the process of isotropization of the universe is studied through the passage time. Moreover from the theoretical point of view anisotropic, universe have a greater generality than isotropic models . The simplicity of the field equations made Bianchi space time useful in constructing models spatially homogeneous and anisotropic cosmologies.

Viscosity is important for number of reasons. Heller and Klimek [2] have investigated viscous fluid cosmological model without initial singularity. They have shown that the introduction of bulk viscosity effectively remove the initial singularity .Roy and Singh [3] have investigated LRS Bianchi Type-V cosmological model with viscosity. Banerjee and Sanyal [4] have investigated Bianchi Type-V cosmological models with viscosity and heat flow. Shri Ram [5] have investigated the field equations for LRS Bianchi Type-I perfect fluid space time by reducing the field equation ti a Riccati equation and new exact solutions are obtained starting from Bayin and Krisch [6]

In the present paper , we have studied Bianchi type- III bulk viscous string cosmological model in general theory of relativity. In the presence of bulk viscosity, to get determinant models we have assume the equation of state $\rho = \lambda$.

2. Field Equation :

We consider string cosmology for the spherically symmetric homogeneous space-time with metric

$$ds^2 = -dt^2 + A^2dr^2 + B^2e^{-2qx}dY^2 + C^2dz^2 \quad (1)$$

where A , B,C are function of t only and q is constant .

In a co-moving coordinate system we set

$u^i = u_i = (1,0,0,0)$

Also, is taken along the direction of

$\chi^i = (0, A^{-1},0,0)$

The Einstein field equation for a cloud of string with bulk viscosity are

$$R_j^i - \frac{1}{2} R g_j^i = \rho \mu^j \mu_i - \lambda \chi^j \chi_i \xi \theta (\mu^j \mu_i + g_j^i) \qquad (2)$$

Where ρ is the rest energy density for a cloud of string with particles attached to them, λ is string tension density and ξ is the coefficient of bulk viscosity of bulk viscosity. Thus we have

$$\rho = \rho_p + \lambda \qquad (3)$$

Here ρ_p is the particles energy density, u_i is the four velocity for a cloud of particles and χ_i is the four vector representing the strings direction which essentially is the direction of anisotropy.

Thus $u^i u_i = -1 = -\chi_i \chi^i$ and $u_i \chi^i = 0$ `11 (4)

The non-vanishing components of the Einstein field equation are

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{q^2}{A^2} = \rho \qquad (5)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{q^2}{A^2} = \lambda + \xi\theta \qquad (6)$$

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = \xi\theta \qquad (7)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = \xi\theta \qquad (8)$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0 \qquad (9)$$

where the suffix 4 at the symbols A,B and C denotes ordinary differentiation with respect to 't' Equations (9) leads to

$$A = \mu B \qquad (10)$$

where μ is an integrating constant.

Using (10) in field equations (5-8) yields

$$2\frac{C_4}{C}\frac{B_4}{B} - \frac{q^2}{\mu^2 B^2} + \frac{B_4^{\ 2}}{B^2} = \rho \qquad (11)$$

$$2\frac{B_{44}}{B} - \frac{q^2}{\mu^2 B^2} + \frac{B_4^{\ 2}}{B^2} = \lambda + \xi\theta \qquad (12)$$

$$\frac{C_{44}}{C} + \frac{B_{44}}{B} + \frac{C_4}{C}\frac{B_4}{A} = \xi\theta \qquad (13)$$

Here, we have three field equation connecting five unknown quantities a, b, ρ, λ, ξ Therefore, in order to obtain exact solution we must need two more relations connecting the unknown quantities. Therefore we assume the equation of state of Nambus as

$$\rho = \lambda \qquad (14)$$

Using equation (14) in (11) and (13), we obtain

$$3\frac{A_4}{A}\frac{B_4}{B} - \frac{B_{44}}{B} + \frac{A_{44}}{A} = 0 \qquad (15)$$

To solve equation we introduce new function R and S given by (Shri Ram, 1987)

$$R = \frac{C_4}{C} \text{ and } S = \frac{B_4}{B} \qquad (16)$$

By using of equation (16), equation (15) becomes

$$3RS - S^2 - S_4 + R^2 + R_4 = 0 \qquad (17)$$

The nonlinear equation (17) can be regarded Riccati equation $S(orR)$

If we solve (17) as Riccati equation is S . it can be linear zed by means of change of function

$$S = S_0 + \frac{1}{z} \qquad (18)$$

Using equation (18) in (17), we get

$$Z_4 + (3R - 2S_0)Z = 1 \qquad (19)$$

Here, S_0 is a particular solution of (17)

$$Z = \frac{M_0^{\ 2}}{C^3}\int C^2 M_0^{\ -2} dt + k_1 \qquad (20)$$

k_1 being integration constant.

Equation(19) and (20) yield

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$$B = B_0 \exp\left[\int \frac{dt}{\frac{M_0^2}{C^3}\left[\frac{C^3}{M_0^2} dt + k_1\right]} + k_2 \right] \qquad (21)$$

k_2 being integration constant. Thus, from the couple $[M_0, C]$ we can generate a new $[B, C]$ where B is given by (21) and C remains invariable.

If (17) is regarded as a Riccati equation in R it can be linear zed by the changed of function

$$R = R_0 + \frac{1}{Y} \qquad (22)$$

Introducing (22) into (17) we obtain

$$Y_4 + (-3S - 2R_0)Y = 1 \qquad (23)$$

where R_0 is particular solution of(17) , from equation (23)

$$Y = B^3 L_0^2 \int \frac{1}{B^3 L_0^2} dt + k_3 \qquad (24)$$

k_3 being interesting constant, from equation (22) and (23), we obtain

$$C = L_0 \exp\left[\int \frac{dt}{B^3 L_0^2 \int \frac{dt}{B^3 A_0^2} + k_3} + k_4 \right] \qquad (25)$$

k_4 is integrating constant. Hence from metric function $[L_0, B]$ we can generate new function $[C, B]$ where C is given by (25) and B remains invariable.

3. Generation of New Exact Solution:

We now apply our generation technique to Bayin and Krisch [6] solution (cf. equation (2.21) and (2.22)) i.e.

$$C = A_1 t + A_2; \qquad M_0 = m \qquad (26)$$

where A_1, A_2 and m are constants. The solution contains three parameters two of which can be transformed away using the coordinates freedom. Therefore, equation (26) is written as

$$C = t \quad ; \qquad M_0 = m \qquad (27)$$

Inserting equation (27)into (21) we obtain

$$C = t, B = \frac{t^4}{4} + k, k_2 = \log k_2, \quad A = \mu B \qquad (28)$$

By change of scale, the metric of equation (28) is

$$ds^2 = -dt^2 + \mu\left(\frac{t^4}{4} + k\right)^2 dx^2 + \left(\frac{t^4}{4} + k\right)^2 e^{-2qx} dy^2 + t^2 dz^2 \qquad (29)$$

k being a constant Applying again equation (21) for the model , we arrive at the metric (29) with different parameter. Inserting equation (26) into (25) as the particular solution we arrive at the metric function (29) with different Parameters. Thus equation (25) does not lead to any new solution.

4. Physical and kinematics properties of equation:

The rest energy (ρ), string tension density (λ), expansion (θ), proper volume (R^3), shear (θ) and bulk viscosity coefficient are given by

$$\rho = \frac{3t^6 + 4t^2 k - 2\frac{q^2}{\mu^2}}{2\left(\frac{t^4}{4} + k\right)^2} = \lambda \qquad (30)$$

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = \frac{9t^4 + 4k}{t(t^4 + 4k)} \qquad (31)$$

$$\sigma^2 = \frac{2}{3}\left[\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{A_4 C_4}{AC}\right] = \frac{2}{3}\left(\frac{3t^4 - 4k}{t(t^4 + 4k)}\right)^2 \qquad (32)$$

$$\xi = \frac{16t^3}{9t^2 + 4k} \qquad (33)$$

$$\frac{\sigma}{\theta} = \frac{\sqrt{2}}{\sqrt{3}}\left(\frac{3t^4 - 4k}{9t^4 + 4k}\right) \qquad (34)$$

$$R^3 = \left(\frac{t^5}{16} + \frac{t^5 k}{2} + tk^2\right) \qquad (35)$$

5. Solution for absence of bulk viscosity:

Then the field equations (11), (12) (13) becomes

$$2\frac{C_4}{C}\frac{B_4}{B} - \frac{q^2}{\mu^2 B^2} + \frac{B_4^2}{B^2} = \rho \qquad (36)$$

$$2\frac{B_{44}}{B}-\frac{q^2}{\mu^2B^2}+\frac{{B_4}^2}{B^2}=\lambda \quad (37)$$

$$\frac{C_{44}}{C}+\frac{B_{44}}{B}+\frac{C_4}{C}\frac{B_4}{A}=0 \quad (38)$$

For equation (38) we introduce new function R and S given by

$$R=\frac{A_4}{A} \quad and \quad S=\frac{B_4}{B} \quad (39)$$

By use of equation (39), equation (38) becomes

$$S_4+S^2+R_4+R^2+RS=0 \quad (40)$$

The nonlinear equation (40) can be regarded a Riccati equation in S (or R)

If we solve (40) as a Riccati equation in S, it can be linear zed by means of change of function

$$S=S_0+\frac{1}{Z} \quad (41)$$

Using equation (41) in (40) we get

$$Z_4+(-2S_0-R)Z=1 \quad (42)$$

where S is a particular solution of (40), equation (42) is a linear first order differential equation which has the general solution

$$Z=CM_0^2\left(\int\frac{1}{M_0^2C}dt+k_1\right) \quad (43)$$

k_1 being an integration constant. Equation (42) and (43) yield

$$B=M_0\exp\left(\int\frac{dt}{M_0^2C\left[\int\frac{dt}{M_0^2C}+k_1\right]}+k_2\right) \quad (44)$$

K_2 is integration constant. Thus, from the couple $[M_0,C]$we can generate new one $[B,C]$ where B is given by (44) and A remain invariable.

If (40) is regarded as a riccati equation in R it can be linear zed by the change of function

$$R=R_0+\frac{1}{Y} \quad (45)$$

Introducing (45) into (40) we obtain

$$Y_4+(-2R_0-S)Z=1 \quad (46)$$

where R_0 is a particular solution of (40) and equation (46) on integration gives

$$Y=BA_0^2\left(\int\frac{1}{BA_0^2}dt+k_3\right) \quad (47)$$

$k_3$being an integration constant, from equation (45) and (47) we obtain

$$C=L_0\exp\left(\int\frac{dt}{L_0^2B\left[\frac{dt}{L_0^2B}+k_3\right]}+k_4\right) \quad (48)$$

k_4 is an integration constant, hence from metric function $[L_0,B]$we can generate new function $[B,C]$ where C is given by (48) and B remain invariable.

6. Generation of new exact solutions:

We now apply our generation technique to Bayin and Krisch (1986) solution (cf. equation (2.21) and (2.22)) i.e.

$$C=A_1t+A_2 \quad ; \quad B_0=m \quad (49)$$

where A_1, A_2 and C are constant. The solution contain three parameter two of which can transformed away using the coordinate freedom. Therefore, equation (42) written as

$$C=t \quad ; \quad M_0=m \quad (50)$$

Inserting equation (50) into (42), we obtain

$$A=t,\ B=ck_2\left[\log t+k_1c^2\right], \quad k_2=\log k_2,$$

$$A=\mu B \quad (51)$$

By the change of scale, the metric of equation (44) is

$$ds^2=-dt^2+\mu^2(\log t+k)^2dx^2+(\log t+k)^2dy^2+t^2dz^2 \quad (52)$$

k being a constant, applying again equation (42) for the model , we arrive at the metric (52) with different parameter. Inserting equation (50) into (48) as the particular solution we arrive at the metric function (52) with different parameters. Thus the equation (48) does not lead to any new solution.

7. Physical and Kinematical properties of equation

The rest energy(ρ), the string tension density(λ), expansion (θ), proper volume (R^3) and shear $(\sigma)$are given by

$$\rho = \frac{t^2 + 2(\log t + k) - 2\frac{q^2}{\mu^2}}{t^2(\log t + k)^2} \qquad (53)$$

$$\lambda = \frac{t^2 - 2(\log t + k) - 2\frac{q^2}{\mu^2}}{t^2(\log t + k)^2} \qquad (54)$$

$$\sigma = \frac{\sqrt{2}}{\sqrt{3}}\frac{1}{t}\left[\frac{1}{\log t + k} - 1\right] \qquad (55)$$

$$\theta = \frac{1}{t}\left[1 + \frac{2}{\log t + k}\right] \qquad (56)$$

$$\frac{\sigma}{\theta} = \frac{\sqrt{2}}{\sqrt{3}}\left[\frac{1 - (\log t + k)}{2 + (\log t + k)}\right] \qquad (57)$$

$$R^3 = \mu t(\log t + k)^2 \qquad (58)$$

8. Conclusion:

In this paper we have obtained of the field of equation for the string cosmological Bianchi type-III space time in the presence and absence of bulk viscosity. The solution in the presence of bulk viscosity are corresponds to the geometric string $\rho = \lambda$ Here , we observe that the model satisfies energy condition $\rho > 0$, $\lambda > 0$ and the bulk viscosity coefficient ξ is positive and ρ and λ vanishes at $t \to \infty$. Since $\lim_{t \to \infty} \frac{\sigma}{\theta} \neq 0$ the model does not approach to isotropy for large value of t. The proper volume R^3 tend to zero when $t \to 0$ and R^3 is infinite when $t \to \infty$.

The model is established in the absence of bulk viscosity. In this case also the model satisfied condition $\rho > 0$, $\lambda > 0$ and ρ, λ vanishes at $t \to \infty$. As gradually increases θ and σ decreases and finally vanish when $t \to \infty$. Also since $\lim_{t \to \infty} \frac{\sigma}{\theta} \neq 0$ the model is not isotropic for large t. The proper volume R^3 tend to zero when $t \to 0$ and is infinite when $t \to \infty$.

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